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Effect of tips of wings moving at supersonic speed

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Effect of Tips of Wings Moving at Supersonic Speed

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Let us consider the problem, linearized in the usual way [1-3], of the vibration of a thin, deformable, finite-span wing moving at supersonic speed. At any point of the

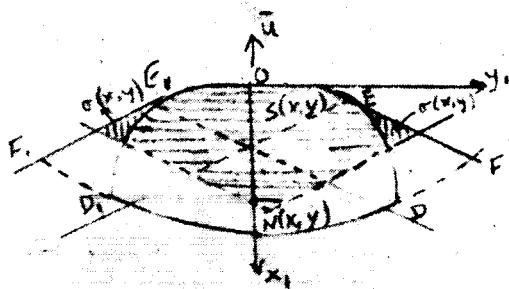


Figure 1.

wing in an x_1, y_1, z_1 coordinate system advancing in a straight line with the basic wing-velocity u , the velocity potential Φ_1 , specified by the vibration,

is represented by formula (6) of reference [4].

Let us consider the wing which has the leading edge defined by $x_1 = \Psi(y_1)$ and the trailing edge by $x_1 = \chi(y_1)$. The trailing-edge function $\chi(y_1)$ satisfies the condition $|d\chi(y_1)/dy_1| \leq \cot \alpha$, where α is the Mach angle. It is assumed here that the normal velocity, specified by the vibration, is given on the wing by $v_n = \partial\Phi_1/\partial z_1 = Re A_1(x_1, y_1) e^{i\omega t}$, where the function $A_1(x_1, y_1)$ defines the form of the oscillations.

In order to calculate the potential Φ_1 at any point of this wing by means of formula (6), it is necessary to find the value of the normal velocity $\partial\Phi_1/\partial z_1$ in the regions $E_1 D_1 F_1$ and $E_1 U_1 F_1$ (figure 1). We transform to x, y, z coordinates

$$\begin{aligned} x &= x_1 - x_{10} - \sqrt{u^2/c^2 - 1} (y_1 - y_{10}) \\ y &= x_1 - x_{10} + \sqrt{u^2/c^2 - 1} (y_1 - y_{10}) \\ z &= \sqrt{u^2/c^2 - 1} z_1. \end{aligned} \quad (1)$$

The origin is placed at $E(x_{10}, y_{10})$ on the leading edge. The point E is so defined that to its left on the leading edge the condition $|\partial \Psi(y_1)/\partial y_2| \leq \cot \alpha$ is fulfilled; on its right this condition is violated. Formula (6) becomes *

$$\varphi = \frac{-e^{\frac{1}{2}\beta(x+y)}}{2\pi s(x,y)} \iint \left[\frac{\partial \psi}{\partial z} \right]_{z=0} e^{-\frac{1}{2}\beta(\xi+m)} \frac{\cos[\lambda\sqrt{(x-\xi)(y-m)}]}{\sqrt{(x-\xi)(y-m)}} d\eta d\xi \quad (7)$$

where $\beta = -\frac{i\omega u}{u^2 - a^2}$, $\lambda = \frac{\omega a}{u^2 - a^2}$, a is the speed of sound in the gas at rest.

The velocity potential at any point $N(x,y)$ of the wing is

$$\begin{aligned} \varphi &= \frac{-e^{\frac{1}{2}\beta(x+y)}}{2\pi s(x,y)} \iint \frac{A(\xi,m) \cos[\lambda\sqrt{(x-\xi)(y-m)}]}{\sqrt{(x-\xi)(y-m)}} d\eta d\xi \\ &- \frac{e^{\frac{1}{2}\beta(x+y)}}{2\pi s(x,y)} \iint \left[\frac{\partial \psi}{\partial z} \right]_{z=0} e^{-\frac{1}{2}\beta(\xi+m)} \frac{\cos[\lambda\sqrt{(x-\xi)(y-m)}]}{\sqrt{(x-\xi)(y-m)}} d\eta d\xi \end{aligned} \quad (7)$$

where the function given on the wing is

$$\begin{aligned} A(x,y) &= e^{-\frac{1}{2}\beta(x+y)} A_1 \left[\frac{x+y + x_{10}}{2} ; \frac{y-x}{2\sqrt{u^2/a^2 - 1}} + y_{10} \right] \frac{e^{i\omega t}}{\sqrt{u^2/a^2 - 1}} \\ &= \frac{\partial \psi}{\partial z} e^{-\frac{1}{2}\beta(x+y)} \end{aligned} \quad (4)$$

Putting the velocity potential ψ , for the point $N(x,y)$ in EDF, which is zero in EDF and $E_1 D_1 F_1$ in (7), we arrive at the integral equation which $\partial \psi / \partial z$ satisfies:

$$\iint \frac{A(\xi,m) \cos[\lambda\sqrt{(x-\xi)(y-m)}]}{\sqrt{(x-\xi)(y-m)}} d\eta d\xi = f(x,y) \quad (5)$$

* The sign \leq is to be understood in all the following formulae.

where $\Theta(x,y)$ denotes the value of $\frac{\partial \varphi}{\partial z} e^{-\frac{1}{2}\beta(x+y)}$ in the

EDF (or $E_1 D_1 F_1$) regions. The variable of integration in the $\sigma(x,y)$ region varies between $0 \leq \xi \leq x$; $\psi(x) \leq \eta \leq y$, where

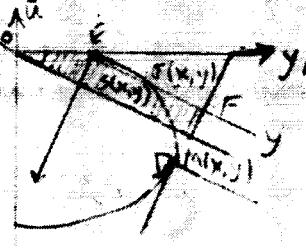


Figure 2.

$y = \psi(x)$ is the equation of arc ED of the tip in the transformed coordinates. The known function $f(x,y)$ is defined as:

$$f(x,y) = - \int_{s(x,y)}^x \frac{A(\xi,\eta) \cos[\lambda \sqrt{(x-\xi)(y-\eta)}]}{\sqrt{(x-\xi)(y-\eta)}} d\eta \quad (6)$$

The regions $e(x,y)$ and $\sigma(x,y)$ are shown in figure 2.

Let us consider the steady motion of a thin, arbitrarily cambered wing. In this case, $\omega = 0$ must be put in all the expressions and instead of condition (4) on the wing must be inserted $-u\beta_0$, where β_0 is the angle of attack of a wing element.

Putting $\lambda = 0$ in (5), and reducing the obtained equation to two Abel equations, keeping in mind the boundary condition $f(0,y) = 0$, we obtain the solution

$$\Theta(x,y) = \frac{1}{\pi^c} \left\{ \frac{1}{\sqrt{y - \psi(x)}} \int_0^x \frac{f_E[\xi, \psi(x)]}{\sqrt{x - \xi}} d\xi + \int_y^\infty \int_x^\infty \frac{f_{E\eta}(\xi, \eta)}{\psi(x) \sqrt{(x-\xi)(y-\eta)}} d\xi \right\} \quad (7)$$

The normal velocity goes to infinity on the arc ED , as $R^{-\frac{1}{2}}$ where R is the distance from $M(x,y)$ to ED .

We assume now that the Mach cones with vertices at E and E_1 intersect the leading edge as shown in figure 3.

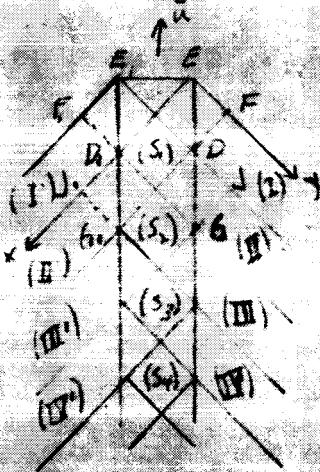


Figure 3

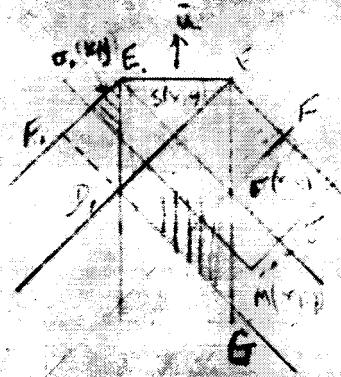


Figure 4

In the regions (I) and (I'), the normal velocity is determined by the solution (7). On the basis of this solution, the velocity potential φ may be calculated by formula (3) for the region (S_1) on the wing lying beyond the Mach cones with vertices at D and D_1 of the tips.

In order to compute the potential on the wing in the region (S_2) lying within the Mach cones with vertices at D and D_1 and beyond the cones with vertices at C and G_1 , it is necessary to find $\partial\varphi/\partial z$ in DGI (or $D_1G_1I_1$).

For the point $M(x,y)$ lying in DGI , we express the potential φ which equals zero in regions (I), (II), (III) (or (I'), (II'), (III')) respectively, by formula (2), dividing the region of integration into three parts as shown in figure 4: $\sum = \alpha + \sigma_1 + \sigma$.

The function $\partial\varphi/\partial z = A(x,y)$ is given on the wing in the region $s(x,y)$. The function $\partial\varphi/\partial z = \alpha(x,y)$ is determined

by the solution (7) for the region $\sigma_1(x, y)$. We denote

$\partial\Phi/\partial z$ by $\Theta_1(x, y)$ in the region $\sigma(x, y)$. Then we arrive at the integral equation which $\Theta_1(x, y)$ satisfies:

$$\iint \frac{\Theta_1(\xi, \eta)}{\sigma(x, y) \sqrt{(x-\xi)(y-\eta)}} d\eta d\xi = \Phi(x, y) \quad (8)$$

The variable of integration in $\sigma(x, y)$ varies between the limits $0 \leq \xi \leq x$, $\psi(\xi) \leq \eta \leq y$. The known function is

$$\Phi(x, y) = f(x, y) + f_1(x, y)$$

where

$$f = - \iint \frac{\Lambda(\xi, \eta)}{s(x, y) \sqrt{(x-\xi)(y-\eta)}} d\eta d\xi$$

$$f_1 = - \iint \frac{\Theta(\xi, \eta)}{\sigma(x, y) \sqrt{(x-\xi)(y-\eta)}} d\eta d\xi \quad (9)$$

Equation (8) differs from the equation to which (5) is reduced for $\lambda=0$ only in the form of the known function. Taking into account the boundary condition $\Phi(0, y) = 0$, we obtain the solution of (8) by using the solution (7) as a ready formula if only $f(x, y)$ is replaced by $\Phi(x, y)$.

Noting that the function f_1 and therefore $f_{1\xi}$ and $f_{1\xi\eta}$ tend identically to zero in the region $\sigma(x, y)$ for $0 \leq x \leq l_1$, where l_1 is the distance between the generators of the Mach cones with vertices at D and G, we obtain the solution of (7) as

$$\Theta_1(x, y) = \frac{1}{\pi^2} \left\{ \frac{1}{\sqrt{y - \psi(x)}} \int_0^x \frac{r_E[\xi, \psi(x)]}{\sqrt{x-\xi}} d\xi + \int_0^x \int_0^y \frac{f_{1\xi\eta}(\xi, \eta)}{\psi(x) \sqrt{(x-\xi)(y-\eta)}} d\eta d\xi \right\} \quad (10)$$

$$+ \frac{1}{\pi^2} \left\{ \frac{1}{\sqrt{y - \psi(x) l_1}} \int_{l_1}^x \frac{f_{1\xi}[\xi, \psi(x)]}{\sqrt{x-\xi}} d\xi + \int_{l_1}^x \int_{l_1}^y \frac{f_{1\xi\eta}(\xi, \eta)}{\psi(x) \sqrt{(x-\xi)(y-\eta)}} d\eta d\xi \right\}$$

The first component coincides with the solution (7). The second component takes into account the effect on the point $\chi(x,y)$ of the opposite tip of the wing $E_1 D_1$.

Reasoning in the same way, we find the value of $\partial\varphi/\partial z$ in the strips (III) and (III'), (IV) and (IV')... Thus, for example, the normal derivative $\partial\varphi/\partial z = \theta_2(x,y)$ for points of (III) is found by means of (10) if instead of substituting $\theta(x,y)$ for f_1 in (9) we substitute $\theta_1(x,y)$, etc.

Therefore, the velocity potential φ may be computed by formula (3) everywhere on the wing in the regions $(S_1), (S_2), (S_3), (S_4)$, etc.

Figures 3 and 4 are given only for the rectangular wing because of its simplicity. The problem may be solved for any form of the tips ED and $E_1 D_1$. In particular, for the rectangular wing of arbitrary aspect ratio, but $\Psi(x) = x$ and $t_1 = \sqrt{2}L$, where L is the semi-span, in (7) and (10).

Let us note that (7) and (10) are correct in those cases when the tip ED is defined not by one equation $y = \Psi(x)$ but consists of curved strips given by the equations $y = \Psi_k(x)$ where $k = 1, 2, \dots, n$.

The above method for determining the normal velocity component may be generalized to the case of wing vibrations.

Actually, the integral equations which the functions $\frac{\partial\varphi}{\partial z} e^{-iz\beta(x+y)}$ satisfy in the regions (I), (II), (III), ... have the form (5) and differ from each other only in the form of the function on the right side. This function is determined from the analogous

case of the steady motion and depends, in the N-th region,
on the solution of the integral equation in the (N-1)-st
region.

The inversion of (5) is given in a later note.

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